# Optimal Threshold Social Choice Algorithm Picks Utilitarian Winner

In an election, the **utilitarian winner** (**UW**) is the candidate or set of candidates who would maximize the collective utility of the voters if they won.

Claude Hillinger, **The Case for Utilitarian Voting. Discussion Papers in Economics 2005-11** : "Utilitarian voting (UV) is defined in this paper as any voting rule that allows the voter to rank all of the alternatives by means of the scores permitted under a given voting scale. Specific UV rules that have been proposed are approval voting, allowing the scores 0, 1; range voting, allowing all numbers in an interval as scores; evaluative voting, allowing the scores -1, 0, 1. The paper deals extensively with Arrow's impossibility theorem that has been interpreted as precluding a satisfactory voting mechanism. I challenge the relevance of the ordinal framework in which that theorem is expressed and argue that instead utilitarian, [i.e.] cardinal social choice theory is relevant for voting. I show that justifications of both utilitarian social choice and of majority rule can be modified to derive UV. The most elementary derivation of UV is based on the view that no justification exists for restricting voters' freedom to rank the alternatives on a given scale."

From my paper, **Proving Social Choice Possible**: "Lehtinen (**A Welfarist Critique of Social Choice Theory: Interpersonal Comparisons in the Theory of Voting.** Erasmus Journal for Philosophy and Economics, 8(2), 34–83) has shown that 'strategic behavior increases the frequency with which the utilitarian winner is chosen compared to sincere behavior '. The utilitarian winner is the one that maximizes the social utility of the social choice. Therefore, the mechanism described in this paper should accomplish two things: sincere voting behavior on the part of individuals and increased selection of the utilitarian winner or winners compared to other voting systems."

Proof Sketch

Consider the winning set, $W = \{w_1, w_2, ..., w_m\}$, which consists of the m highest vote getters. The set, $Y = \{y_1, y_2, ..., y_n\}$, orders the candidates by the number of votes received by each candidate. $y_1Ry_2R ... Ry_n$. Consider the candidate in the winning set who has the least number of votes, $y_m$. Call the set of voters who voted for this candidate set A. Also consider the next highest ranked candidate, $y_{m+1}$ who has at most $y_m$ -1votes. Call the set of voters who voted for this candidate set B. The question is is it possible for the voters in set B to have greater total utility than the voters in set A? Could we replace $y_m$ in the winning set with $y_{m-1}$ and have greater total utility? Or at least would a tie between $y_m$ and $y_{m-1}$ result in higher social utility? This means that the utility of set B would have to be greater than or equal to the utility of set A. The intersection of sets A and B, A∩B, contains voters who voted for both candidates $y_m$ and $y_{m-1}$. Their collective utility for candidate $y_{m-1}$ could be greater than their collective utility for candidate $y_m$. However, since we have no knowledge of the actual utilities, we assume a fully randomized distribution which would not give an advantage in collective utility to either voters for $y_m$ or voters for $y_{m-1}$ in A∩B. However, if a voter voted for $y_m$ but not for $y_{m-1}$, we know that their utility for $y_m$ is greater than their utility for $y_{m-1}$. Also, if a voter voted for $y_{m-1}$ but not for $y_m$, their utility for $y_{m-1}$ is greater than their utility for $y_m$. Therefore, the collective utility of the set A - A∩B for $y_m$ would be greater than the collective utility of this set for $y_{m-1}$. Also the collective utility of the set B - A∩B for $y_{m-1}$ is greater than the collective utility of this set for $y_m$ since all the voters in this set voted for $y_{m-1}$ but not for $y_m$. We know that there is at least one more voter in set A - A∩B than in the set B - A∩B since $y_m R y_{m-1}$. Therefore, the utility of the set, $W = \{w_1, w_2, ..., w_m\}$ is greater than the utility of the set, $W = \{w_1, w_2, ..., w_{m-1}, w_{m+1}\}$, and the set W represents the utilitarian winner.